Algebra 2

Lesson 5-7

Example 1 Determine Number and Type of Roots
Solve each equation. State the number and type of roots.
a. $9x - 4 = 0$

$9x - 4 = 0$	Original equation
$9x = 4$	Add 4 to each side.
$x = \frac{4}{9}$	Divide each side by 9.

This equation has exactly one real root, $\frac{4}{9}$.

b. $6x^3 - 10x^2 + 16x = 0$

$6x^3 - 10x^2 + 16x = 0$	Original equation
$2x(3x^2 - 5x + 8) = 0$	Factor out the GCF.

One root is 0 since $2x = 0$ by the Zero Product Property. However, $3x^2 - 5x + 8$ is not factorable, so use the Quadratic Formula to find the roots of that trinomial.

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$	Quadratic Formula
$x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(3)(8)}}{2(3)}$	Substitute 3 for a, –5 for b, and 8 for c.
$= \frac{5 \pm \sqrt{-71}}{6}$	Simplify.
$= \frac{5 \pm i\sqrt{71}}{6}$	$\sqrt{-1} = i$

This equation has one real root, 0, and two imaginary roots, $\frac{5 - i\sqrt{71}}{6}$ and $\frac{5 + i\sqrt{71}}{6}$.

Example 2 Find Numbers of Positive and Negative Zeros
State the possible number of positive real zeros, negative real zeros, and imaginary zeros of $h(x) = -3x^6 + 4x^4 + 2x^2 - 6$.

Since $h(x)$ has degree 6, it has six zeros. However, some of them may be imaginary. Use Descartes' Rule of Signs to determine the number and type of real zeros. Count the number of changes in sign for the coefficients of $h(x)$.

$h(x) = \quad -3x^6 \quad + \quad 4x^4 \quad + \quad 2x^2 \quad - \quad 6$

yes	no	yes
– to +	+ to +	+ to –

Since there are two sign changes, there are 2 or 0 positive real zeros.

Find $h(-x)$ and count the number of changes in signs for its coefficients.

$h(-x) = -3(-x)^6 + 4(-x)^4 + 2(-x)^2 - 6$

$= -3x^6 + 4x^4 + 2x^2 - 6$

yes	no	yes
– to +	+ to +	+ to –

Since there are two sign changes, there are 2 or 0 negative real zeros.

Thus, the function $h(x)$ has either 2 or 0 positive real zeros and either 2 or 0 negative real zeros. Make a chart of the possible combinations of real and imaginary zeros.

Number of Positive Real Zeros	Number of Negative Real Zeros	Number of Imaginary Zeros	Total Number of Zeros
2	2	2	2 + 2 + 2 = 6
2	0	4	2 + 0 + 4 = 6
0	2	4	0 + 2 + 4 = 6
0	0	6	0 + 0 + 6 = 6

Example 3 Use Synthetic Substitution to Find Zeros
Find all of the zeros of $g(x) = x^4 + x^3 + 3x^2 + 5x - 10$.

Step 1 Since $g(x)$ has degree 4, the function has four zeros. To determine the possible number and type of real zeros, examine the number of sign changes for $g(x)$ and $g(-x)$.

$g(x) = x^4 \quad + x^3 \quad + 3x^2 \quad + 5x \quad - 10$

no no no yes

$g(-x) = x^4 \quad - x^3 \quad + 3x^2 \quad - 5x \quad - 10$

yes yes yes no

Step 2 Since there is 1 sign change for the coefficients of $g(x)$, the function has exactly one positive real zero. Since there are 3 sign changes for the coefficient of $g(-x)$, $g(x)$ has 3 or 1 negative real zeros. Thus $g(x)$ has either 4 real zeros, or 2 real zeros and 2 imaginary zeros.

Step 3 You can see that there is only one positive real zero. Begin by testing some easy possibilities such as 1. You can use synthetic substitution to determine whether $x - 1$ is a factor of the polynomial.

1	1	1	3	5	–10
		1	2	5	10
	1	2	5	10	0

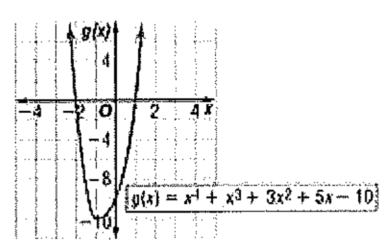
Since the remainder is 0, $x - 1$ is a factor of the polynomial. The polynomial can be factored as $(x - 1)(x^3 + 2x^2 + 5x + 10)$. The polynomial $x^3 + 2x^2 + 5x + 10$ is the depressed polynomial. Check to see if this polynomial can be factored.

$(x - 1)(x^3 + 2x^2 + 5x + 10)$ Factored polynomial
$(x - 1)[x^2(x + 2) + 5(x + 2)]$ Factor the depressed polynomial by grouping.
$(x - 1)(x^2 + 5)(x + 2)$

Use the Zero Product Property to find all the zeros of the function.

$x^4 + x^3 + 3x^2 + 5x - 10 = 0$ Original equation
$(x - 1)(x + 2)(x^2 + 5) = 0$ Factor.
$x - 1 = 0$ or $x + 2 = 0$ or $x^2 + 5 = 0$ Zero Product Property
$x = 1$ $\quad x = -2$ $\quad x^2 = -5$ Solve each equation.
$x = \pm\sqrt{-5}$
$x = \pm i\sqrt{5}$

Thus the function has one real positive zero, one real negative zero, and two imaginary zeros. The zeros are -2, 1, $i\sqrt{5}$, and $-i\sqrt{5}$.



The graph of the function verifies that there are two real zeros.

Example 4 Use Zeros to Write a Polynomial Function
Write a polynomial function of least degree with integral coefficients whose zeros include –1, 5, and 3 + *i*.

Understand If $3 + i$ is a zero, then $3 - i$ is also a zero according to the Complex Conjugates Theorem. So, $x + 1$, $x - 5$, $x - (3 + i)$, and $x - (3 - i)$ are factors of the polynomial function.

Plan Write the polynomial function as a product of its factors.

$$f(x) = (x + 1)(x - 5)[x - (3 + i)][x - (3 - i)]$$

Solve Multiply the factors to find the polynomial function.

$f(x) = (x + 1)(x - 5)[x - (3 + i)][x - (3 - i)]$	Write an equation.
$= (x + 1)(x - 5)[(x - 3) - i][(x - 3) + i]$	Regroup terms.
$= (x^2 - 4x - 5)[(x - 3) - i][(x - 3) + i]$	Multiply the first two factors.
$= (x^2 - 4x - 5)[(x - 3)^2 - i^2]$	Rewrite as the difference of two squares.
$= (x^2 - 4x - 5)[(x^2 - 6x + 9) - (-1)]$	Square $x - 3$ and replace i^2 with -1.
$= (x^2 - 4x - 5)[x^2 - 6x + 10]$	Simplify.
$= x^4 - 6x^3 + 10x^2 - 4x^3 + 24x^2 - 40x - 5x^2 + 30x - 50$	Multiply using the Distributive Property.
$= x^4 - 10x^3 + 29x^2 - 10x - 50$	Combine like terms.

$f(x) = x^4 - 10x^3 + 29x^2 - 10x - 50$ is a polynomial function of least degree with integral coefficients whose zeros are -1, 5, $3 + i$, and $3 - i$.