

Algebra 2

Lesson 5-6

Example 1 Synthetic Substitution

If $f(x) = 5x^3 + 2x^2 - x + 4$, find $f(-1)$.

Method 1 Synthetic Substitution

By the Remainder Theorem, $f(-1)$ should be the remainder when you divide the polynomial by $x + 1$.

$$\begin{array}{r|rrrr} -1 & 5 & 2 & -1 & 4 \\ & & -5 & 3 & -2 \\ \hline & 5 & -3 & 2 & 2 \end{array}$$

The remainder is 2. Thus, by using synthetic substitution, $f(-1) = 2$.

Method 2 Direct Substitution

Replace x with -1 .

$$\begin{array}{ll} f(x) = 5x^3 + 2x^2 - x + 4 & \text{Original function} \\ f(-1) = 5(-1)^3 + 2(-1)^2 - (-1) + 4 & \text{Replace } x \text{ with } -1. \\ = -5 + 2 + 1 + 4 \text{ or } 2 & \text{Simplify.} \end{array}$$

By using direct substitution, $f(-1) = 2$. Both methods give the same result.

Real-World Example 2 Find Function Values

ALGEBRA Show that $2x - 1$ is a factor of $6x^3 + 23x^2 - 5x - 4$. Then find the remaining factors of the polynomial.

Use division to rewrite the divisor so it has a first coefficient of 1.

$$\frac{(6x^3 + 23x^2 - 5x - 4) \div 2}{(2x - 1) \div 2} = \frac{3x^3 + \frac{23}{2}x^2 - \frac{5}{2}x - 2}{x - \frac{1}{2}}$$

The binomial $x - \frac{1}{2}$ is a factor of the polynomial if $\frac{1}{2}$ is a zero of the polynomial function. Use the Factor Theorem and synthetic division.

$$\begin{array}{r|rrrrr} \frac{1}{2} & 3 & \frac{23}{2} & -\frac{5}{2} & -2 & \\ & & \frac{3}{2} & \frac{13}{2} & 2 & \\ \hline & 3 & 13 & 4 & 0 & \end{array}$$

Since the remainder is 0, $2x - 1$ is a factor of the polynomial. The polynomial $6x^3 + 23x^2 - 5x - 4$ can be factored as $(2x - 1)(3x^2 + 13x + 4)$. The polynomial $3x^2 + 13x + 4$ is the depressed polynomial. Check to see if this polynomial can be factored.

$$3x^2 + 13x + 4 = (3x + 1)(x + 4) \quad \text{Factor the trinomial.}$$

$$\text{So, } 6x^3 + 23x^2 - 5x - 4 = (2x - 1)(3x + 1)(x + 4).$$

Example 3 Use the Factor Theorem

Show that $2x - 1$ is a factor of $6x^3 + 23x^2 - 5x - 4$. Then find the remaining factors of the polynomial.

Use division to rewrite the divisor so it has a first coefficient of 1.

$$\frac{(6x^3 + 23x^2 - 5x - 4) \div 2}{(2x - 1) \div 2} = \frac{3x^3 + \frac{23}{2}x^2 - \frac{5}{2}x - 2}{x - \frac{1}{2}}$$

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$$3x^2 + 13x + 4 = (3x + 1)(x + 4) \quad \text{Factor the trinomial.}$$

$$\text{So, } 6x^3 + 23x^2 - 5x - 4 = (2x - 1)(3x + 1)(x + 4).$$