Algebra 2

Lesson 5-5

Example 1 Sum and Difference of Cubes
Factor each polynomial. If the polynomial cannot be factored, write *prime*.

a. $108x^4y + 500xy^4$

$108x^4y + 500xy^4 = 4y(27x^3 + 125y^3)$ — Factor out the GCF.

$27x^3$ and $125y^3$ are both perfect cubes, so we can factor the sum of two cubes.

$$\begin{aligned} 27x^3 + 125y^3 &= (3x)^3 + (5y)^3 && (3x)^3 = 27x^3;\ (5y)^3 = 125y^3 \\ &= (3x + 5y)[(3x)^2 - (3x)(5y) + (5y)^2] && \text{Sum of two cubes} \\ &= (3x + 5y)(9x^2 - 5xy + 25y^2) && \text{Simplify.} \\ 108x^4y + 500xy^4 &= 4xy(3x + 5y)(9x^2 - 15xy + 25y^2) && \text{Replace the GCF.} \end{aligned}$$

b. $64m^3 + 7n^3$

The first term is a perfect cube, but the second term is not. So, the polynomial cannot be factored using the sum of two cubes pattern. The polynomial also cannot be factored using quadratic methods or the GCF. Therefore, it is a prime polynomial.

Example 2 Factoring by Grouping
Factor each polynomial. If the polynomial cannot be factored, write *prime*.

a. $-18ax^3 - 12bx^2 + 6x^2$

$$\begin{aligned} -18ax^3 - 12bx^2 + 6x^2 &= (-1 \cdot 2 \cdot 3 \cdot 3 \cdot a \cdot x \cdot x \cdot x) + (-1 \cdot 2 \cdot 2 \cdot 3 \cdot b \cdot x \cdot x) + (2 \cdot 3 \cdot x \cdot x) \\ &= (6x^2 \cdot -3ax) + (6x^2 \cdot -2b) + (6x^2 \cdot 1) && \text{The GCF is } 6x^2. \\ &= 6x^2(-3ax - 2b + 1) && \text{The remaining polynomial cannot be factored.} \end{aligned}$$

b. $x^3 + 5x^2 - 7x - 35$

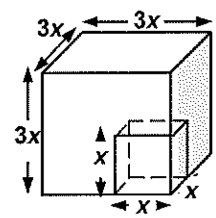
$$\begin{aligned} x^3 + 5x^2 - 7x - 35 &= (x^3 + 5x^2) + (-7x - 35) && \text{Group to find a GCF.} \\ &= x^2(x + 5) + (-7)(x + 5) && \text{Factor the GCF of each binomial.} \\ &= (x + 5)(x^2 - 7) && \text{Distributive Property} \end{aligned}$$

Example 3 Combine Cubes and Squares
Factor each polynomial. If the polynomial cannot be factored, write *prime*.

a. $z^3 - 8x^3$
$z^3 = (z)^3$ and $(2x)^3 = 8x^3$. Thus, this is the difference of two cubes.
$z^3 - 8x^3 = (z - 2x)[z^2 + z(2x) + (2x)^2]$ Difference of two cubes formula with $a = z$ and $b = 2x$
$= (z - 2x)(z^2 + 2zx + 4x^2)$ Simplify.

b. $m^3y^2 - 4m^3y + 4m^3 - n^3y^2 + 4n^3y - 4n^3$
With six terms, factor by grouping first.
$m^3y^2 - 4m^3y + 4m^3 - n^3y^2 + 4n^3y - 4n^3$
$= (m^3y^2 - 4m^3y + 4m^3) + (-n^3y^2 + 4n^3y - 4n^3)$ Group to find a GCF.
$= m^3(y^2 - 4y + 4) - n^3(y^2 - 4y + 4)$ Factor the GCF.
$= (m^3 - n^3)(y^2 - 4y + 4)$ Distributive Property
$= (m - n)(m^2 + mn + n^2)(y^2 - 4y + 4)$ Difference of cubes
$= (m - n)(m^2 + mn + n^2)(y - 2)^2$ Perfect squares

Real-World Example 4 Solve Polynomial Functions by Factoring
GEOMETRY If the small cube is one-third the length of the larger cube and the volume of the figure is 3250 cubic centimeters, what should be the dimensions of the cubes?



Since the length of the smaller cube is one-third the length of the larger cube, then their lengths can be represented by x and $3x$, respectively. The volume of the object equals the volume of the larger cube minus the volume of the smaller cube.

$(3x)^3 - x^3 = 3250$	Volume of object
$27x^3 - x^3 = 3250$	$(3x)^3 = 27x^3$
$26x^3 = 3250$	Subtract.
$x^3 = 125$	Divide.
$x^3 - 125 = 0$	Subtract 125.
$(x)^3 - 5^3 = 0$	Write in cubic form.
$(x - 5)(x^2 + 5x + 25) = 0$	Difference of cubes
$x - 5 = 0$ or $x^2 + 5x + 25 = 0$	Zero Product Property
$x = 5$ $\quad x = \dfrac{-5 \pm 5i\sqrt{3}}{2}$	

Since 5 is the only real solution, the lengths of the cubes are 5 cm and 15 cm.

Example 5 Quadratic Form
Write each expression in quadratic form, if possible.

a. $x^6 + 2x^3 + 5$

$x^6 + 2x^3 + 5 = (x^3)^2 + 2(x^3) + 5 \qquad (x^3)^2 = x^6$

b. $3x^9 - 4x^3 + 1$

This cannot be written in quadratic form since $x^9 \neq (x^3)^2$.

Example 6 Solve Equations in Quadratic Form
Solve $x^4 - 26x^2 + 25 = 0$**.**

$x^4 - 26x^2 + 25 = 0$	Original equation
$(x^2)^2 - 26(x^2) + 25 = 0$	Write the expression on the left in quadratic form.
$(x^2 - 1)(x^2 - 25) = 0$	Factor the trinomial.
$(x-1)(x+1)(x-5)(x+5) = 0$	Factor each difference of squares.

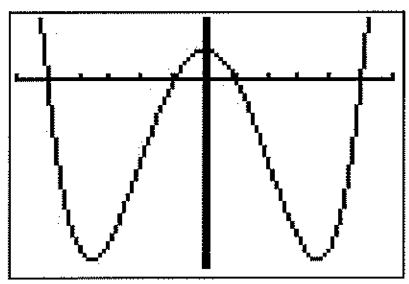
Use the Zero Product Property.

$(x-1) = 0$ or $(x+1) = 0$ or $(x-5) = 0$ or $(x+5) = 0$

$x = 1$ $\quad x = -1$ $\quad x = 5$ $\quad x = -5$

The solutions are -5, -1, 1, and 5.

Check The graph of $f(x) = x^4 - 26x^2 + 25$ shows that the graph intersects the x-axis at $-5, -1, 1$, and 5.



Window: (x = -6 – 6; scale=1), (y = -150 – 50; scale = 1)