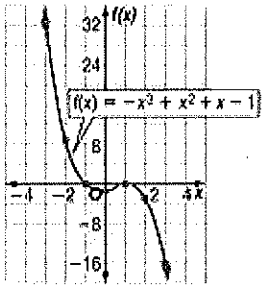
Algebra 2

Lesson 5-4

Example 1 Graph of Polynomial Functions
Graph the polynomial function $f(x) = -x^3 + x^2 + x - 1$ by making a table of values.

Make a table for several values of x and plot the points. Connect the points with a smooth curve.

x	$f(x)$
–3	32
–2	9
–1	0
0	–1
1	0
2	–3
3	–16

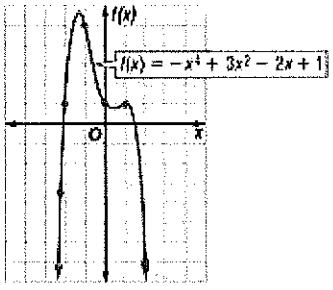


This is an odd–degree polynomial with a negative leading coefficient, so $f(x) \to -\infty$ as $x \to +\infty$, and $f(x) \to +\infty$ as $x \to -\infty$. Notice that the graph intersects the x–axis at two points, indicating there are two real zeros for this function.

Example 2 Locate Zeros of a Function
Determine the values of x between which the real zeros of the function $f(x) = -x^4 + 3x^2 - 2x + 1$ are located. Then draw the graph.

Make a table of values. Since $f(x)$ is a fourth–degree polynomial function, it could have up to 4 real zeros. Look at the values of $f(x)$ to locate the zeros. Then use the points to sketch a graph of the function.

x	$f(x)$	
–3	–47	
–2	1	change in signs
–1	5	
0	1	
1	1	
2	–7	change in signs
3	–59	

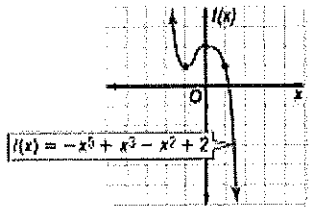


The changes in sign indicate that there are zeros between $x = -3$ and $x = -2$, and between $x = 1$ and $x = 2$.

Example 3 Maximum and Minimum Points
Graph $f(x) = -x^5 + x^3 - x^2 + 2$. Estimate the x–coordinates at which the relative maxima and relative minima occur.

Make a table of values and graph the equation.

x	$f(x)$	
–3	209	
–2	22	
–1	1	← indicates a relative minimum
0	2	← indicates a relative maximum
1	1	
2	–26	zero between 1 and 2
3	–223	



Look at the table of values and the graph.

- The values of $f(x)$ change signs between $x = 1$ and $x = 2$, indicating a zero of the function.
- The value of $f(x)$ at $x = -1$ is less than the surrounding points, so it is a relative minimum.
- The value of $f(x)$ at $x = 0$ is greater than the surrounding points, so it is a relative maximum.

Real-World Example 4 Graph a Polynomial Model

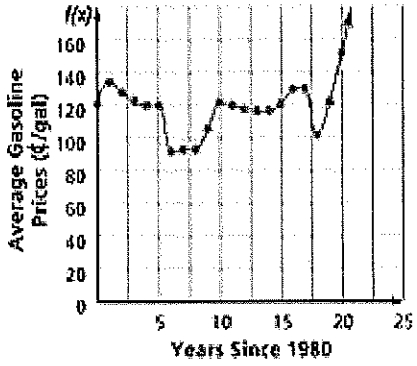
TRAVEL The table shows the average retail prices of gasoline from 1980 through 2000. The x–values are given as years since 1980, so that 1980 = 0, 1981 = 1, and so on, and the values of $f(x)$ are the prices.

x	$f(x)$	x	$f(x)$	x	$f(x)$	x	$f(x)$
0	122	5	120	10	122	15	121
1	135	6	93	11	120	16	129
2	128	7	96	12	119	17	129
3	123	8	96	13	117	18	112
4	120	9	106	14	117	19	122
						20	153

Source: ***World Almanac***

a. Graph the function.

Plot the points from the table.



b. Describe the turning points of the graph and its end behavior.

There are relative maxima at 1, 10, 16–17, and 20. There are relative minima at 6, 13–14, and 18. For the end behavior, as x is increasing, $f(x)$ is increasing, but only from 1999 to 2000.

c. What do the maxima and minima represent in this situation?

The maxima are high prices for gasoline in a particular time period and the minima are low prices for gasoline in a particular time period.