Algebra 2

Lesson 5-3

Example 1 Degrees and Leading Coefficients
State the degree and leading coefficient of each polynomial in one variable. If it is not a polynomial in one variable, explain why.

a. $-2a^3 + a - \dfrac{1}{a^2}$

This is not a polynomial. The term $\dfrac{1}{a^2}$ cannot be written in the form x^n, where n is a nonnegative integer.

b. $b^5 + b^2 - 8$

This is a polynomial in one variable. The degree is 5, and the leading coefficient is 1.

c. $11 - 5c + 2c^6 - 4c^3$

Rewrite the expression so the powers of c are in decreasing order.

$$2c^6 - 4c^3 - 5c + 11$$

This is a polynomial in one variable. The degree is 6, and the leading coefficient is 2.

Real-World Example 2 Evaluate a Polynomial Function
ALGEBRA In a polygon with n sides, for $n > 3$, the number of diagonals can be found by using the function $d(n) = \frac{1}{2}n^2 - \frac{3}{2}n$, where $d(n)$ is the number of diagonals and n is the number of sides. Find the number of diagonals for a decagon (a ten–sided polygon).

$d(n) = \frac{1}{2}n^2 - \frac{3}{2}n$	Given function
$d(10) = \frac{1}{2}(10)^2 - \frac{3}{2}(10)$	Replace n with 10.
$= 50 - 15$	Evaluate.
$= 35$	Simplify.

Example 3 Function Values of Variables
Find $-3[q(4b-1)]$ if $q(x) = x^2 - x + 1$.

First, evaluate $q(4b-1)$ by replacing x in $q(x)$ with $4b-1$.

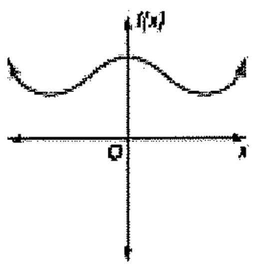
$q(x) = x^2 - x + 1$	Original function
$q(4b-1) = (4b-1)^2 - (4b-1) + 1$	Replace x with $4b-1$.
$= 16b^2 - 8b + 1 - 4b + 1 + 1$	Evaluate $(4b-1)^2$ and $-1(4b-1)$.
$= 16b^2 - 12b + 3$	Simplify.

To evaluate $-3[q(4b-1)]$, multiply $q(4b-1)$ by -3.

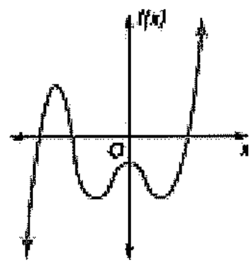
$-3[q(4b-1)] = -3(16b^2 - 12b + 3)$	Substitute $16b^2 - 12b + 3$ for $q(4b-1)$.
$= -48b^2 + 36b - 9$	Distributive Property

Example 4 Graphs of Polynomial Functions
For each graph,
- **describe the end behavior.**
- **determine whether it represents an odd–degree or an even–degree polynomial function, and**
- **state the number of real zeros.**

a.



b.



a.
- As $x \to +\infty, f(x) \to +\infty$. As $x \to -\infty, f(x) \to +\infty$.
- It is an even–degree polynomial function.
- The graph does not intersect the x–axis, so it has no real zeros.

b.
- As $x \to +\infty, f(x) \to +\infty$. As $x \to -\infty, f(x) \to -\infty$.
- It is an odd–degree polynomial function.
- The graph intersects the x–axis at three points, so the function has three real zeros.