Algebra 2

Lesson 5-2

Example 1 Divide a Polynomial by a Monomial

Simplify $\dfrac{25a^5b^3 - 20a^4b^2 + 10a^3b}{5a^2b}$.

$\dfrac{25a^5b^3 - 20a^4b^2 + 10a^3b}{5a^2b} = \dfrac{25a^5b^3}{5a^2b} - \dfrac{20a^4b^2}{5a^2b} + \dfrac{10a^3b}{5a^2b}$ Sum of quotients.

$= \dfrac{25}{5} \cdot a^{5-2}b^{3-1} - \dfrac{20}{5} \cdot a^{4-2}b^{2-1} + \dfrac{10}{5} \cdot a^{3-2}b^{1-1}$ Divide.

$= 5a^3b^2 - 4a^2b + 2a$ $b^{1-1} = b^0$ or 1

Example 2 Division Algorithm

Use long division to find $(x^2 - 7x - 18) \div (x + 2)$.

$$
\begin{array}{r l}
x - 9 & \\
x + 2 \overline{) x^2 - 7x - 18} & \\
(-)\ \underline{x^2 + 2x} & \text{Multiply divisor by } x. \\
-9x - 18 & \text{Subtract. Bring down next term.} \\
(-)\ \underline{-9x - 18} & \text{Multiply divisor by 9.} \\
0 & \text{Subtract.}
\end{array}
$$

The quotient is $x - 9$. The remainder is 0.

Standardized Test Example 3
Which expression is equal to $(m^2 + m - 35)(m - 5)^{-1}$?

A. $m + 6 - \frac{5}{m - 5}$ **B.** $m + 6$ **C.** $m - 6$ **D.** $m - 6 - \frac{5}{m - 5}$

Read the Test Item

Since the second factor has an exponent of –1, this is a division problem.

$$(m^2 + m - 35)(m - 5)^{-1} = \frac{m^2 + m - 35}{m - 5}$$

Solve the Test Item

$$\begin{array}{r} m + 6 \\ m - 5 \overline{\smash{)}\, m^2 + m - 35} \\ \underline{(-)\, m^2 - 5m} \\ 6m - 35 \\ \underline{(-) 6m - 30} \\ -5 \end{array}$$

$m(m - 5) = m^2 - 5m$
$m - (-5m) = 6m$
$6(m - 5) = 6m - 30$
Subtract. $-35 - (-30) = -5$

The quotient is $m + 6$, and the remainder is –5. Therefore,

$$(m^2 + m - 35)(m - 5)^{-1} = m + 6 - \frac{5}{m - 5}.$$

The answer is A.

Example 4 Synthetic Division
Use synthetic division to find $(x^4 + 3x^3 - 2x + 5) \div (x + 1)$.

Step 1 Write the terms of the dividend so that the degrees of the terms are in descending order. Then write just the coefficients as shown at the right. Since there is no x^2 term, you must include a coefficient of 0 for x^2.

$$\begin{array}{ccccc} x^4 & +3x^3 & +0x^2 & -2x & +5 \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ 1 & 3 & 0 & -2 & 5 \end{array}$$

Step 2 Write the constant r of the divisor $x - r$ to the left. In this case, $r = -1$. Bring the first coefficient, 1, down as shown.

$$\begin{array}{r|rrrrr} -1 & 1 & 3 & 0 & -2 & 5 \\ & & & & & \\ \hline & 1 & & & & | \end{array}$$

Step 3 Multiply the first coefficient by r: $1 \cdot -1 = -1$. Write the product under the second coefficient. Then add the product and the second coefficient: $3 + (-1) = 2$.

$$\begin{array}{r|rrrrr} -1 & 1 & 3 & 0 & -2 & 5 \\ & & -1 & & & \\ \hline & 1 & 2 & & & | \end{array}$$

Step 4 Multiply the sum, 2, by r: $-1(2) = -2$. Write the product under the next coefficient and add: $0 + (-2) = -2$.

$$\begin{array}{r|rrrrr} -1 & 1 & 3 & 0 & -2 & 5 \\ & & -1 & -2 & & \\ \hline & 1 & 2 & -2 & & | \end{array}$$

Step 5 Multiply the sum, -2, by r: $-1(-2) = 2$. Write the product under the next coefficient and add: $-2 + 2 = 0$.

$$\begin{array}{r|rrrrr} -1 & 1 & 3 & 0 & -2 & 5 \\ & & -1 & -2 & 2 & \\ \hline & 1 & 2 & -2 & 0 & | \end{array}$$

Step 6 Multiply the sum, 0, by r: $-1(0) = 0$. Write the product under the next coefficient and add: $5 + 0 = 5$. The remainder is 5.

$$\begin{array}{r|rrrrr} -1 & 1 & 3 & 0 & -2 & 5 \\ & & -1 & -2 & 2 & 0 \\ \hline & 1 & 2 & -2 & 0 & |\,5 \end{array}$$

The numbers along the bottom row are the coefficients of the quotient. Start with the power that is one less than the degree of the dividend.

Thus, the quotient is $x^3 + 2x^2 - 2x + \frac{5}{x+1}$.

Example 5 Divisor with First Coefficient Other than 1
Use synthetic division to find $(12x^3 - 7x^2 + 4x - 3) \div (3x - 1)$.

Use division to rewrite the divisor so it has a first coefficient of 1.

$$\frac{12x^3 - 7x^2 + 4x - 3}{3x - 1} = \frac{(12x^3 - 7x^2 + 4x - 3) \div 3}{(3x - 1) \div 3}$$

Divide the numerator and denominator by 3.

$$= \frac{4x^3 - \frac{7}{3}x^2 + \frac{4}{3}x - 1}{x - \frac{1}{3}}$$

Simplify numerator and denominator.

$x - r = x - \frac{1}{3}$, so $r = \frac{1}{3}$.

$$\begin{array}{c|cccc} \frac{1}{3} & 4 & -\frac{7}{3} & \frac{4}{3} & -1 \\ & & \frac{4}{3} & -\frac{1}{3} & \frac{1}{3} \\ \hline & 4 & -1 & 1 & -\frac{2}{3} \end{array}$$

The result is $4x^2 - x + 1 - \dfrac{\frac{2}{3}}{x - \frac{1}{3}}$. Now simplify the fraction.

$$\frac{\frac{2}{3}}{x - \frac{1}{3}} = \frac{2}{3} \div \left(x - \frac{1}{3}\right)$$

Rewrite as a division expression.

$$= \frac{2}{3} \div \frac{3x - 1}{3}$$

$x - \frac{1}{3} = \frac{3x}{3} - \frac{1}{3} = \frac{3x - 1}{3}$

$$= \frac{2}{3} \cdot \frac{3}{3x - 1}$$

Multiply by the reciprocal.

$$= \frac{2}{3x - 1}$$

Multiply.

The solution is $4x^2 - x + 1 - \frac{2}{3x - 1}$.

Check Divide using long division.

$$\begin{array}{r} 4x^2 - x + 1 \\ 3x - 1 \overline{)12x^3 - 7x^2 + 4x - 3} \\ \underline{(-)\ 12x^3 - 4x^2} \\ -3x^2 + 4x \\ (-)\ \underline{-3x^2 + x} \\ 3x - 3 \\ \underline{(-)\ 3x - 1} \\ -2 \end{array}$$

The result is $4x^2 - x + 1 - \frac{2}{3x - 1}$. ✓