Algebra 2

Lesson 5-1

Example 1 Simplify Expressions
Simplify each expression.
a. $(-5a^5x)(-9a^3x^7)$.

$$(-5a^5x)(-9a^3x^7) = (-5 \cdot a \cdot a \cdot a \cdot a \cdot a \cdot x)(-9 \cdot a \cdot a \cdot a \cdot x \cdot x \cdot x \cdot x \cdot x \cdot x \cdot x)$$ Definition of exponents

$$= (-5)(-9) \cdot a \cdot a \cdot a \cdot a \cdot a \cdot a \cdot a \cdot a \cdot x \cdot x \cdot x \cdot x \cdot x \cdot x \cdot x \cdot x$$ Commutative Property

$$= 45a^8x^8$$ Definition of exponents

b. $(x^{-2})(x^6y^{-4})(z^{-3})$

$$(x^{-2})(x^6y^{-4})(z^{-3}) = \left(\frac{1}{x^2}\right)\left(\frac{x^6}{y^4}\right)\left(\frac{1}{z^3}\right)$$ Definition of negative exponents

$$= \left(\frac{1}{x \cdot x}\right)\left(\frac{x \cdot x \cdot x \cdot x \cdot x \cdot x}{y \cdot y \cdot y \cdot y}\right)\left(\frac{1}{z \cdot z \cdot z}\right)$$ Definition of exponents

$$= \left(\frac{1}{\cancel{x} \cdot \cancel{x}}\right)\left(\frac{\cancel{x} \cdot \cancel{x} \cdot x \cdot x \cdot x \cdot x}{y \cdot y \cdot y \cdot y}\right)\left(\frac{1}{z \cdot z \cdot z}\right)$$ Cancel out common factors

$$= \frac{x^4}{y^4z^3}$$ Definition of exponents and fractions

c. $\dfrac{-15c2d}{3cd4}$ **, assume that** $c, d \neq 0$.

$$\frac{-15c^2d}{3cd^4} = \left(\frac{-15}{3}\right) \cdot c^{2-1}d^{1-4}$$ Subtract exponents.

$$= -5cd^{-3} \text{ or } \frac{-5c}{d^3}$$ Remember that a simplified expression cannot contain negative exponents.

Example 2 Degree of a Polynomial
Determine whether each expression is a polynomial. If it is a polynomial, state the degree of the polynomial.

a. $\frac{1}{3}b^2 + \frac{4}{b} - 2$

This expression is not a polynomial because the term $\frac{4}{b}$ is not a monomial.

b. $an^3 - an + \frac{2}{5}n$

This expression is a polynomial because each term is a monomial. The degree of the first term is 1 + 3 or 4, the degree of the second term is 1 + 1 or 2, and the degree of the third term is 1. The degree of the polynomial is 4.

Example 3 Simplify Polynomial Expressions
Simplify each expression.
a. $(-4a^3 + a^2 - 1) - (-3a^3 - a^2 + 2a + 5)$

$$\begin{aligned}(-4a^3 + a^2 - 1) - (-3a^3 - a^2 + 2a + 5) &= -4a^3 + a^2 - 1 + 3a^3 + a^2 - 2a - 5 \\ &= (-4a^3 + 3a^3) + (a^2 + a^2) - 2a + (-1 - 5) \\ &= -a^3 + 2a^2 - 2a - 6\end{aligned}$$

Distribute the −1.
Group like terms.
Combine like terms.

b. $(3a^2 - a - 6) + (-a^2 - a + 3)$

Align like terms vertically and add.

$$\begin{array}{r} 3a^2 - a - 6 \\ (+)\quad -a^2 - a + 3 \\ \hline 2a^2 - 2a - 3 \end{array}$$

Example 4 Simplify by Using the Distributive Property
Find $-2b(3b^3 - b^2 + 5b - 6)$.

$$\begin{aligned}-2b(3b^3 - b^2 + 5b - 6) &= -2b(3b^3) + (-2b)(-b^2) + (-2b)(5b) + (-2b)(-6) \\ &= -6b^4 + 2b^3 - 10b^2 + 12b\end{aligned}$$

Distributive Property
Multiply the monomials.

Real-World Example 5 Write a Polynomial Expression

TRAVEL A charter bus company will allow their drivers to work between periods of rest to eight hours. For the first part of a shift, Davis drives at a speed of 60 miles per hour, and for the second part of the shift, he drives at a speed of 75 miles per hour. Write a polynomial to represent the distance driven.

Words	60 mph for some time, and 75 mph for the rest
Variable	Let x = the number of hours he drives at 60 miles per hour.
Expression	$60x + 75(8 - x)$

$60x + 75(8 - x)$	Original expression
$= 60x + 600 - 75x$	Distributive Property
$= 600 - 15x$	Combine like terms.

The polynomial is $600 - 15x$.

Example 6 Multiply Polynomials

Find $(m + n)(2m^2 - 5mn + 8n)$.

$(m + n)(2m^2 - 5mn + 8n)$	
$= m(2m^2 - 5mn + 8n) + n(2m^2 - 5mn + 8n)$	Distributive Property
$= m(2m^2) + m(-5mn) + m(8n) + n(2m^2) + n(-5mn) + n(8n)$	Distributive Property
$= 2m^3 - 5m^2n + 8mn + 2m^2n - 5mn^2 + 8n^2$	Multiply monomials.
$= 2m^3 - 3m^2n + 8mn - 5mn^2 + 8n^2$	Combine like terms.